J. KSIAM Vol.20, No.2, 163–174, 2016 http://dx.doi.org/10.12941/jksiam.2016.20.163

PATH AVERAGED OPTION VALUE CRITERIA FOR SELECTING BETTER OPTIONS

JUNSEOK KIM[1], MINHYUN YOO[2], HYEJU SON[3], SEUNGGYU LEE[1], MYEONG-HYEON KIM[4], YONGHO CHOI[1], DARAE JEONG[1], AND YOUNG ROCK KIM[5†]

[1]DEPARTMENT OF MATHEMATICS, KOREA UNIVERSITY, SEOUL 136-713, KOREA

[2]DEPARTMENT OF FINANCIAL ENGINEERING, KOREA UNIVERSITY, SEOUL 136-701, KOREA

[3]DEPARTMENT OF ECONOMICS, SOOKMYUNG WOMEN'S UNIVERSITY, SEOUL 140-742, KOREA

[4]BUSINESS SCHOOL, KOREA UNIVERSITY, SEOUL 136-701, KOREA

[5]MAJOR IN MATHEMATICS EDUCATION, HANKUK UNIVERSITY OF FOREIGN STUDIES, SEOUL 130-791, KOREA

E-mail address: rocky777@hufs.ac.kr

ABSTRACT. In this paper, we propose an optimal choice scheme to determine the best option among comparable options whose current expectations are all the same under the condition that an investor has a confidence in the future value realization of underlying assets. For this purpose, we use a path-averaged option as our base instrument in which we calculate the time discounted value along the path and divide it by the number of time steps for a given expected path. First, we consider three European call options such as vanilla, cash-or-nothing, and asset-or-nothing as our comparable set of choice schemes. Next, we perform the experiments using historical data to prove the usefulness of our proposed scheme. The test suggests that the path-averaged option value is a good guideline to choose an optimal option.

1. INTRODUCTION

When it comes to investment, selecting appropriate financial products and adjusting the ratio of those selected items, i.e., maximizing expected return of the portfolio and minimizing the risk of the investment at the same time, should be one's top priority. There have been many theories concerning standards for compromising the 'ideal' portfolio. 'Portfolio Theory', by Sharpe, is one of the competing theories. To be specific, Sharpe insisted that, to maximize the benefit, investors should find the set of mean and variance of portfolios and select the one that provides the greatest expected utility. Similarly, the idea of comparing the set of mean of different types of options is an underlying base in this paper. When we choose one option

Received by the editors May 27 2016; Revised June 11 2016; Accepted in revised form June 11 2016; Published online June 22 2016.

2000 *Mathematics Subject Classification.* 90B50, 91G60.

Key words and phrases. Black–Scholes equations, European options, path-averaged option value.

† Corresponding author.

under the condition that all the options' current expectations are the same, we have to make a decision as to the best for the investment. In this paper, we present a method of an appropriate judgement for a better option choice.

Until now, there have been various attempts to evaluate option prices in financial markets. Beginning in 1973, it was described that a mathematical framework for finding the fair price of a European option by Black and Scholes [1, 2], several numerical methods have been presented for the cases where analytic solutions are neither available nor easily computable. See more details about numerical methods such as finite difference method (FDM) [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13], finite element method [14, 15, 16], finite volume method [17, 18, 19], and a fast Fourier transform [20, 21, 22, 23, 24]. For convenience, we use the closed-form of the Black–Scholes equation in this work. Next, we will describe the proposed method for choosing the better option in next section. In the proposed method, we compute various path-averaged option values and then choose the option which gives the maximum value.

The rest of the paper is organized as follows. Section 2 describes the proposed method to determine the best option among comparable options whose current expectations are all the same. We present the numerical experiments in Section 3. Finally, conclusions are drawn in Section 4.

2. Proposed method

For simplicity of exposition, we consider European call options. Let x denote the value of underlying asset, t be time, σ be the volatility of return on the underlying asset, and r be the risk-free interest rate. T and E represent the maturity and predetermined exercise price of option, respectively. Let C_1, C_2, and C_3 be the prices of vanilla European, cash-or-nothing, and asset-or-nothing call options, respectively. Equations (2.1)–(2.3) are the payoff functions for those options. In Fig. 1(a), (b), and (c), the solid and dash lines are the payoff at maturity and the value of options at time $t = 0$.

$$\Lambda_1(x) = \max\left(x - E, 0\right), \tag{2.1}$$

$$\Lambda_2(x) = \begin{cases} 0 & \text{if } x < E, \\ K & \text{otherwise,} \end{cases} \tag{2.2}$$

$$\Lambda_3(x) = \begin{cases} 0 & \text{if } x < E, \\ x & \text{otherwise.} \end{cases} \tag{2.3}$$

Similarly, let P_1, P_2, and P_3 be the prices of vanilla European, cash-or-nothing, and asset-or-nothing put options, respectively. Equations (2.4)–(2.6) are also the payoff functions for European put options. In Fig. 2(a), (b), and (c), the solid and dash lines are the payoff at

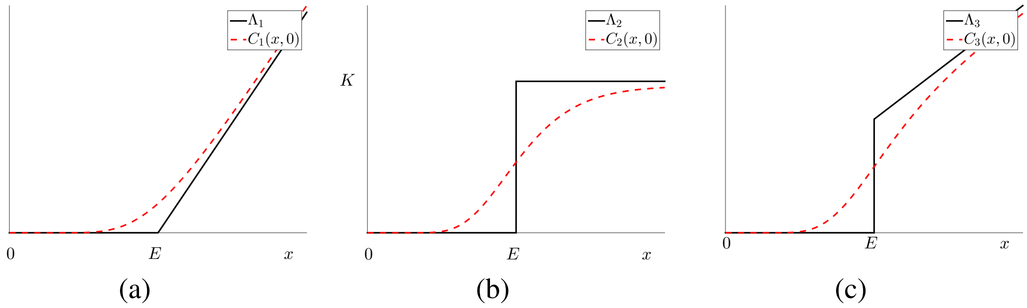


FIGURE 1. Payoffs of (a) European vanilla, (b) cash-or-nothing, and (c) asset-or-nothing call options. The solid and dash lines are the payoff at maturity and the value of options at time $t = 0$.

maturity and the value of options at time $t = 0$.

$$\Lambda_1(x) = \max\left(E - x, 0\right), \tag{2.4}$$

$$\Lambda_2(x) = \begin{cases} K & \text{if } x < E, \\ 0 & \text{otherwise}, \end{cases} \tag{2.5}$$

$$\Lambda_3(x) = \begin{cases} x & \text{if } x < E, \\ 0 & \text{otherwise}. \end{cases} \tag{2.6}$$

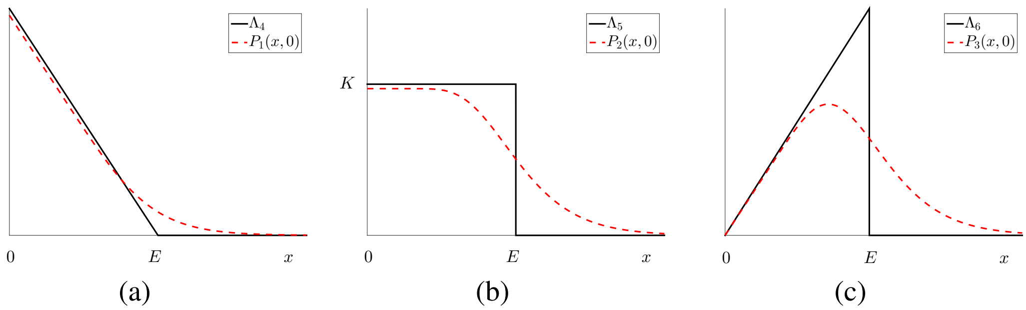


FIGURE 2. Payoffs of (a) European vanilla, (b) cash-or-nothing, and (c) asset-or-nothing put options. The solid and dash lines are the payoff at maturity and the value of options at time $t = 0$.

The closed-form solutions [25, 26] for the European call and put options with payoffs (2.1)–(2.6) are given as follows.

$$C_1(x,t) = xN(d_1) - Ee^{-r(T-t)}N(d_2), \tag{2.7}$$

$$C_2(x,t) = Ke^{-r(T-t)}N(d_2), \tag{2.8}$$

$$C_3(x,t) = xe^{-r(T-t)}N(d_1), \tag{2.9}$$

$$P_1(x,t) = Ee^{-r(T-t)}N(-d_2) - xN(-d_1), \tag{2.10}$$

$$P_2(x,t) = Ke^{-r(T-t)}N(-d_2), \tag{2.11}$$

$$P_3(x,t) = xe^{-r(T-t)}N(-d_1), \tag{2.12}$$

where $N(d) = 1/\sqrt{2\pi}\int_{-\infty}^{d} e^{-0.5x^2}\mathrm{d}x$ is the cumulative distribution function for the standard normal distribution,

$$d_1 = \frac{\log(x/E) + (r + 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}}, \quad d_2 = d_1 - \sigma\sqrt{T-t}.$$

In the proposed method, we use the path-averaged option value (PAOV) which is the average of discounted value of option from each time steps to the present along the path as

$$\text{PAOV} = \sum_{i=1}^{N} \frac{V(x,t_i)e^{-rt}}{N}, \tag{2.13}$$

where $V(x,t)$ is the value of the option at time t and N is the number of days. Here, the time t_i represents a discrete observation time, with uniform time steps. Thus, $t_i - t_{i-1} = h$. Since we assume constant slope on the value of the underlying asset, the PAOV is obtained by the average of discounted option price on day by day.

3. Numerical experiments

In the previous section, we have introduced the concept of PAOV, path averaged option value, to calculate expected value of the option in the aspect of future tendencies of underlying asset. Now we will actually apply this concept both theoretically and practically. In the subsection 3.1, we set certain linear direction of future tendencies of underlying asset. To be specific, we calculate PAOV of 6 options (Vanilla call/put, Cash-or-nothing call/put, and Asset-or-nothing call/put), and compare the theoretical value. In the subsection 3.2, on the other hand, we apply PAOV with real stock data, KOSPI200, in more practical way. We find specific periods of stock data movements that have increase and decrease tendency, and calculate PAOV of them. In this process, we first calculate PAOV with real data of the real date, and then we fit a linear line for each tendencies and calculated PAOV with those fitted data. In addition, since the majority of option market in the real world is consisted with vanilla option, we perform our test only with European vanilla call and put options.

3.1. **Stock path with the drift.** In this section, we present the numerical tests to compare the performance of options with respect to the direction of underlying asset. Equations (2.7)–(2.12) are used to value an European option on a non-dividend stock. The value of underlying asset at present value $x(0)$ and the exercise price E are 100, respectively. The maturity is 1 year, and the number of the time steps is 365. The riskless interest rate r is 3%. We firstly compare PAOV of call options given the condition that the underlying asset has a certain tendency, and then we compare PAOV of put options. The procedure is explained in the next paragraph.

Table 1 represents the call option price at time $t = 0$ and a ratio of the price of the asset-or-nothing to the price of vanilla call option, which means the option buyer takes a long position with an amount of ratio of vanilla call and cash-or-nothing. Moreover, we have set the present value of cash-or-nothing same with that of vanilla call option by adjusting designated cash payoff K. Calculating the ratio and adjusting the present value are to make sure that different kinds of options have same condition.

TABLE 1. The price of the three options at time $t = 0$ and the ratio of asset-or-nothing to vanilla call option values.

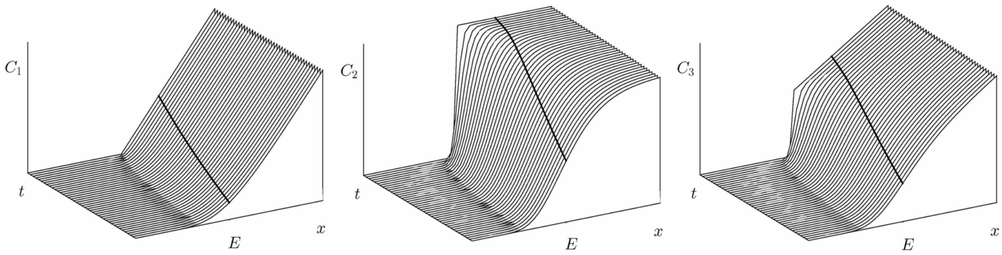
Types	Vanilla call	Cash-or-nothing	Asset-or-nothing	Ratio
	13.2833084	13.2833084	58.1011880	4.374

Table 2 shows the path-averaged call option values with respect to the direction of underlying asset. The obtained ratio is reflected in the calculation of asset-or-nothing option values, i.e., we divide the value of asset-or-nothing by the ratio for the fair comparison. We test every cases with linear increase or decrease at certain percentage which makes linear direction of underlying asset in daily market.

TABLE 2. PAOVs of call options with respect to the direction of underlying asset.

Types	-0.1%	-0.05%	0%	+0.05%	+0.1%
Vanilla call	16.107322	22.950863	37.393584	65.434432	99.379309
Cash-or-nothing	22.685429	33.689913	58.743554	82.868066	93.373250
Asset-or-nothing	21.381259	31.622485	54.799640	80.333005	96.336642

From Table 2, we can observe the fact that PAOVs of cash-or-nothing and asset-or-nothing tend to move together. To be specific, the variation of PAOV between vanilla call and either cash-or-nothing or asset-or-nothing is bigger than that of between cash-or-nothing and asset-or-nothing. This result comes from the different payoff structures of each option. When we check the payoffs of Fig. 3, we can easily notice that both cash-or-nothing and asset-or-nothing have discontinuous payoff at the maturity. Consequently, the values of the option jump up at the time near maturity, resulting in higher PAOV of cash-or-nothing and asset-or-nothing. However, we can also observe that PAOV of vanilla call option is the largest when the underlying asset



(a) (b) (c)

FIGURE 3. The value of European call options with respect to the increasing path of underlying asset. (a) Vanilla European, (b) cash-or-nothing, and (c) asset-or-nothing options.

increases by 0.1%. Table 2 suggests that we should choose options which give the highest PAOV depending on the expected path direction.

Similarly, Table 3 shows the put option price at time $t = 0$ and a ratio of the price of the asset-or-nothing to the price of vanilla put option, which means the option buyer takes a long position with an amount of ratio of vanilla put and cash-or-nothing.

TABLE 3. The price of the options at time $t = 0$ and the ratio of asset-or-nothing to vanilla put option values.

Types	Vanilla put	Cash-or-nothing	Asset-or-nothing	Ratio
	10.3278618	10.3278618	38.9433654	3.771

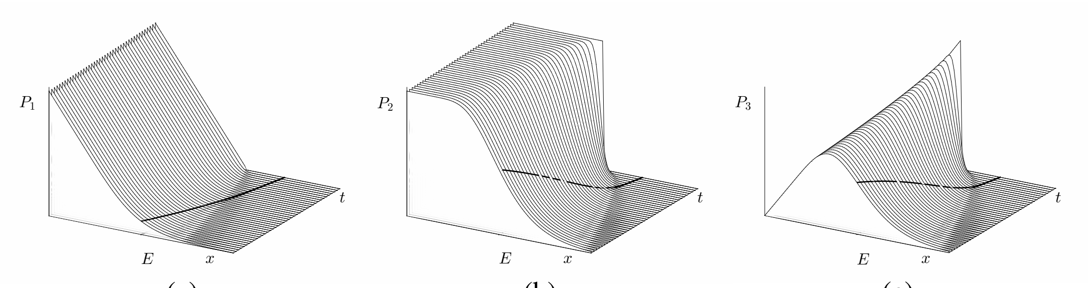
Also, the present value of vanilla put option is set to be same with cash-or-nothing by adjusting the designated cash payoff K, and again, this is our intention to make same comparison condition between different option types. Table 4 shows the path-averaged put option values with respect to the direction of underlying asset.

TABLE 4. PAOV of put options with respect to the direction of underlying asset.

Types	-0.1%	-0.05%	0%	+0.05%	+0.1%
Vanilla put	75.797600	47.969067	26.691586	17.136683	12.671502
Cash-or-nothing	60.863853	54.053611	38.344239	23.619190	17.117946
Asset-or-nothing	57.955672	56.570245	41.983720	25.571325	18.423489

From Table 4, we can observe the fact that PAOV of cash-or-nothing and asset-or-nothing tend to move together again, and vise versa. When we check the payoffs of Fig. 4, we can easily notice that both cash-or-nothing and asset-or-nothing have discontinuous payoff at the maturity. Consequently, the values of the option jumps up at the time near maturity, resulting in higher PAOV of cash-or-nothing and asset-or-nothing. However, we can also observe that

PAOV of vanilla put option is the largest when the underlying asset decreases by 0.1%. To sum up, all these differences are due to different structures of options, and since those structures are easily changed by many factors, we can only calculate and judge the options by PAOV.



(a) (b) (c)

FIGURE 4. The value of European put options with respect to the increasing path of underlying asset. (a) Vanilla European, (b) cash-or-nothing, and (c) asset-or-nothing options.

Next, we consider a PAOV of the path we tested with random perturbation for sensitivity analysis. We generate the path with normal distributed perturbation of mean $\mu = 0$ and standard deviation $\sigma = 0.1$.

The result is almost similar with the experiment without perturbations. See Tables 5 and 6.

TABLE 5. PAOV of call options with respect to the direction with random perturbation of underlying asset.

Types	-0.1%	-0.05%	0%	+0.05%	+0.1%
Vanilla call	16.113370	22.968903	37.428544	65.414449	99.328331
Cash-or-nothing	22.692883	33.721284	58.804893	82.839290	93.319291
Asset-or-nothing	21.379911	31.621394	54.826034	80.332875	96.341435

TABLE 6. PAOV of put options with respect to the direction with random perturbation of underlying asset.

Types	-0.1%	-0.05%	0%	+0.05%	+0.1%
Vanilla put	75.775506	47.903553	26.664359	17.142853	12.678970
Cash-or-nothing	60.834855	53.990866	38.504766	23.629520	17.128893
Asset-or-nothing	57.952913	56.573101	42.228138	25.572186	18.421479

3.2. **Stock path with real data.** In this subsection, we apply the PAOV with real stock data. Here the stock data, KOSPI200 index announced by Korea Securities and Future exchange (KRX), are used [27]. KOSPI200 has existed since 1964, which consists of 200 representative constituents. These constituents were chosen based on the largeness of capital stocks. We can easily understand it as a national index such as S&P 500 of U.S.A. Figure 5 shows the prices of KOSPI200 for 2 years, from August 5, 2013 to August 6, 2015.

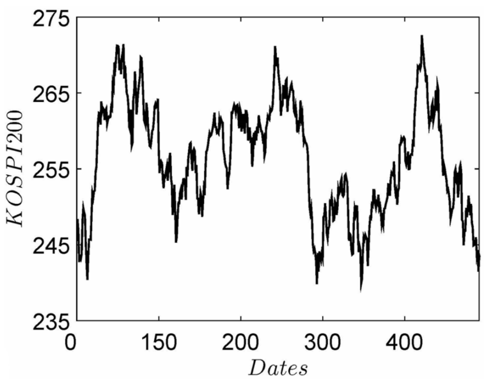


FIGURE 5. The rough variation of KOSPI200 for 2 years, from August 5, 2013 to August 6, 2015.

As a first step, we intend to apply PAOV with the random movement of underlying asset. We select the data from March 2, 2015 to June 1, 2015, that shows random movements, and manipulate them for efficient implementation. Then we calculate PAOV of the vanilla call and put option for KOSPI200 index at different strike prices, using the pricing formula derived by Black and Scholes (see Eqs. (2.7) and (2.10)). A schematic for the evaluating procedure is shown in Figure 6. We perform the test in a similar way on previous test with parameters as follows.

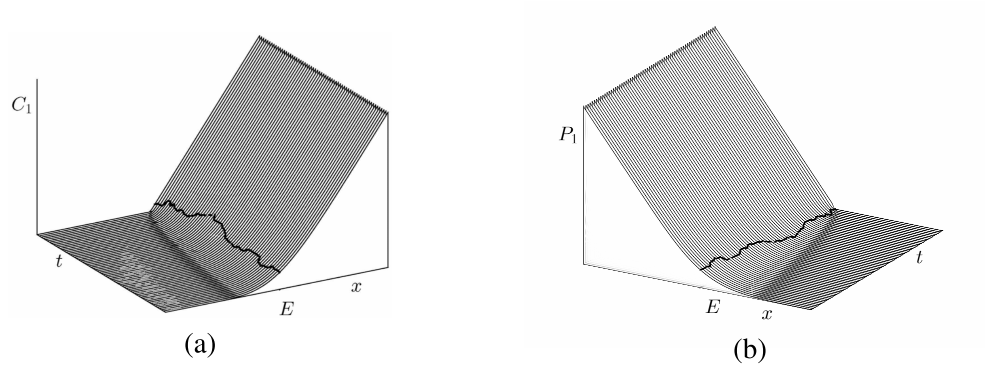


FIGURE 6. Schematic for vanilla (a) call and (b) put options on KOSPI200 data.

The underlying asset is KOSPI200, the maturity is 0.25 with 63 operation days from March $2, 2015$ to June $1, 2015$. The riskless interest rate r is 1.65%, and the volatility on return of underlying asset σ is 0.3. The result is shown in Table 7. As we can infer from the different payoff structures of vanilla call and put options, PAOV of vanilla call option increases when strike price decreases. On the other hand, PAOV of vanilla put option increases as strike price increases.

TABLE 7. PAOVs with respect to different strike price.

Strike price	245	247.5	250	252.5	255	257.5	260
call	20.6673	18.8363	17.0817	15.4107	13.8313	12.3521	11.0014
put	4.5226	5.1813	5.9165	6.7353	7.6457	8.6563	9.7755

Next, we consider the windows for specific dates in real data to reflect a sort of tendency. The criteria for choosing certain tendencies are based on the concept called moving average. Here, moving average is one of the most frequently used indicators of stock market, which reflects long and short term tendency of stock index movements. For instance, if someone tries to calculate 20 days of moving average on a specified date, he or she has to gather data of past 20 operation days and simply divide the sum of data by 20. In the same way, we can easily calculate moving average of a period with specific lengths. 20 and 60 days of moving averages are the typically used periods, which generally imply short and long term of the stock tendency, respectively.

Practically, if 20 days of moving average increases, overtaking 60 days of moving average, this implies that the current stock indexes are on the rise. Thus, this tendency is interpreted as a signal of purchasing stocks. With the decreasing tendency of 20 days of moving average, on the other hand, which passes down through 60 days of moving average, the shareholders are encouraged to sell the stocks. These concepts are known as golden cross and dead cross.

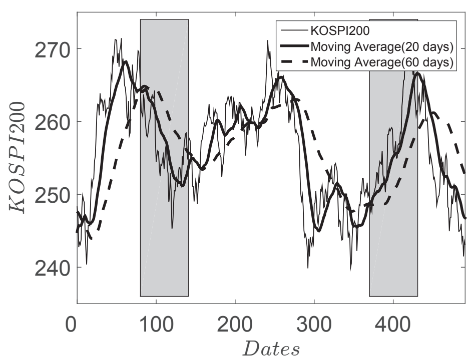


FIGURE 7. Moving average of 20 and 60 days from August 5, 2013 to August 6, 2015.

In the Fig. 7, we calculate both 20 days and 60 days of moving average of KOSPI200, from August $5, 2013$ to August $5, 2015$, with 492 days of operation. By calculating the proportion of 20 days of moving average divided by that of 60's, we select 2 periods that represents decreasing (from December $2, 2013$ to February $2, 2014$) and increasing (from February $2, 2015$ to April $30, 2015$) tendency of KOSPI200, namely, dead cross and golden cross. These periods are shaded in Fig. 8 below. Figure 8 represents some of the dates which we focus on.

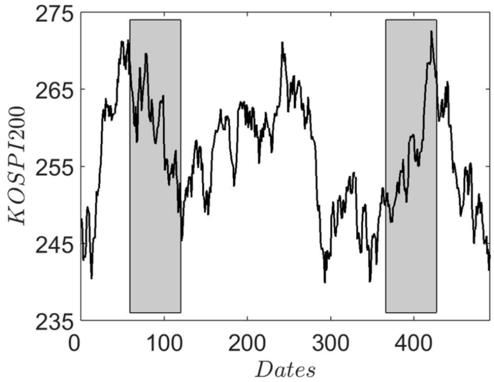


FIGURE 8. Shaded areas which represent the decreasing (left side, the first period) and increasing (right side, the second period) tendency.

Tables 8 and 9 show PAOV from the first period and the second period of KOSPI200 data, respectively.

TABLE 8. PAOV in the first period (November 1, 2013 $\sim$ January 29, 2014) from Fig 8.

Strike	245	247.5	250	252.5	255	257.5	260
put	7.12061	8.34253	9.64627	11.05333	12.55678	14.15021	15.82721

TABLE 9. PAOV in the second period (February 2, 2015 $\sim$ April 30, 2015) from Fig 8.

Strike	245	247.5	250	252.5	255	257.5	260
call	24.85838	22.94389	21.09232	19.30742	17.59279	15.95195	14.38826

First, we obtain the ratios of each strike on the basis of the present option price of median strike (252.5) which is meant to manage the share, making same present option price. Next, we obtain the PAOV and multiply them by the ratios.

Tables 10 and 11 show PAOV on linear tendency from data of the first and second period.

TABLE 10. PAOV on linear decreasing tendency from the first period (November 1, 2013 $\sim$ January 29, 2014) from Fig 8.

Strike	245	247.5	250	252.5	255	257.5	260
put	6.83285	8.02358	9.30524	10.69801	12.19162	13.77698	15.44646

TABLE 11. PAOV on linear increasing tendency from the second period (February 2, 2015 $\sim$ April 30, 2015) from Fig 8.

Strike	245	247.5	250	252.5	255	257.5	260
call	25.73309	23.76498	21.85788	20.01647	18.24550	16.54985	14.93444

In this part, we simply draw a linear line by connecting the first and last KOSPI200 indexes in each periods respectively, and again calculated PAOV of them. We conclude that the results in the Tables 10 and 11 show no significance here, since the variances of PAOV have not changed.

4. CONCLUSION

The simple and useful method was proposed for choosing a better option among options under the conditions that all the options' current expectations are the same and one has a confidence in predicting future tendency of underlying assets. To choose a better option, we compute the PAOV. As test problems, we considered three European call and put options such as vanilla, cash-or-nothing, and asset-or-nothing since there exist closed-form solutions. The proposed method is general. Therefore it can be applied to other options. If there is no closed-form solution is available, then we can use numerical approximations such as finite difference method, finite element method, and Monte Carlo simulation. The test results suggested that the PAOV is a good guideline to choose a better option.

ACKNOWLEDGMENT

The corresponding author (Young Rock Kim) was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education(NRF-2015R1D1A1A01059643) and supported by the National Institute of Mathematics Sciences(NIMS).

REFERENCES

[1] F. Black and M. Scholes, *The pricing of options and corporate liabilities*, J. Polit. Econ., **81** (1973), 637–659.

[2] C. Wang, S. Zhou, and J. Yang, *The pricing of vulnerable options in a fractional Brownian motion environment*, Discrete Dyn. Nat. Soc., **2015** (2015), 579213.

[3] D.J. Duffy, *Finite Difference Methods in Financial Engineering*, John Wiley & Sons, New York, NY, USA, (2006).

[4] H. Han and X. Wu, *A fast numerical method for the Black–Scholes equation of American options*, SIAM J. Numer. Anal., **41**(6) (2003), 2081–2095.

[5] D. Jeong, T. Ha, M. Kim, J. Shin, I.H. Yoon, and J. Kim, *An adaptive finite difference method using far-field boundary conditions for the Black–Scholes equation*, B. Korean Math. Soc., **51**(4) (2014), 1087–1100.
[6] D. Jeong and J. Kim, *A comparison study of ADI and operator splitting methods on option pricing models*, J. Comput. Appl. Math., **247** (2013), 162–171.
[7] D. Jeong, J. Kim, and I.S. Wee, *An accurate and efficient numerical method for Black–Scholes equations*, Commun. Korean Math. Soc., **24**(4) (2009), 617–628.
[8] D. Jeong, I.S. Wee, and J. Kim, *An operator splitting method for pricing the ELS option*, J. KSIAM, **14** (2010), 175–187.
[9] R. Seydel, *Tools for Computational Finance*, Springer, Berlin, Germany, (2003).
[10] D. Tavella and C. Randall, *Pricing Financial Instruments: The Finite Difference Method*, John Wiley & Sons, New York, NY, USA, (2000).
[11] J. Topper, *Financial Engineering with Finite Elements*, John Wiley & Sons, Chichester, UK, (2005).
[12] P. Wilmott, J. Dewynne, and S. Howison, *Option Pricing: Mathematical Models and Computation*, Oxford Financial Press, Oxford, UK, (1993).
[13] D. Jeong, S. Seo, H. Hwang, D. Lee, Y. Choi, and J. Kim, *Accuracy, robustness, and efficiency of the linear boundary condition for the Black–Scholes equations*, Discrete Dyn. Nat. Soc., **2015** (2015) 359028.
[14] Y. Achdou and N. Tchou, *Variational analysis for the Black and Scholes equation with stochastic volatility*, ESAIM-Math. Model. Num., **36**)(3) (2002), 373–395.
[15] A. Ern, S. Villeneuve, and A. Zanette, *Adaptive finite element methods for local volatility European option pricing*, Int. J. Theor. Appl. Financ., **7**(6) (2004), 659–684.
[16] N. Rambeerich, D.Y. Tangman, M.R. Lollchund, and M. Bhuruth, *High-order computational methods for option valuation under multifactor models*, Eur. J. Oper. Res., **224**(1) (2013), 219–226.
[17] P.A. Forsyth and K.R. Vetzal, *Quadratic convergence for valuing American options using a penalty method*, SIAM J. Sci. Comput., **23**(6) (2002), 2095–2122.
[18] S. Wang, *A novel fitted finite volume method for the Black–Scholes equation governing option pricing*, IMA J. Numer. Anal., **24**(4) (2004), 699–720.
[19] S. Wang, S. Zhang, and Z. Fang, *A super convergent fitted finite volume method for Black–Scholes equations governing European and American option valuation*, Numer. Meth. Part. D. E., **31**(4) (2015), 1190–1208.
[20] P. Carr and D.B. Madan, *Option valuation using the fast Fourier transform*, J. Comput. Financ., **2**(4) (1999), 61–73.
[21] A. Cerny, *Introduction to fast Fourier transform in finance*, J. Deriv., **12**(1) (2004), 73–88.
[22] M.A.H. Dempster and S.S.G. Hong, *Spread option valuation and the fast Fourier transform*, Springer Finance, Springer Finance, Springer, Berlin, (2002), 203–220.
[23] C.C. Hsu, S.K. Lin, and T.F. Chen, *Pricing and hedging European energy derivatives: a case study of WTI oil options*, Asia-Pac. J. Financ. St., **43**(3) (2014), 317–355.
[24] T. Sakuma and Y. Yamada, *Application of homotopy analysis method to option pricing under Levy processes*, Asia-Pac. Financ. Mark., **21**(1) (2014), 1–14.
[25] E.G. Haug, *The Complete Guide to Option Pricing Formulas*, New York, McGraw-Hill, (1998).
[26] M. Rubinstein, *One for another*, Risk, **4**(7) (1991), 30–32.
[27] *Korea Exchange, Historical KOSPI 200 index option price*, http://www.krx.co.kr/m3/m3_2/m3_2_1/JHPKOR03002_01.jsp, (2015).